**中山大学本科生期末考试**

**考试科目：《高等代数》（B卷）**

学年学期：2016年第1学期 姓 名：

学 院/系：数据院 学 号：

考试方式：闭卷 年级专业：

考试时长：120分钟 班 别：

任课教师：

警示 《中山大学授予学士学位工作细则》第八条：“考试作弊者，不授予学士学位。”

温馨提示：(1) 计算可以慢点，细心点，每部分最后一题有点难度;(2) 卷中向量均有→在符号的顶部。

(3) 普通班同学可以中英互用;(4) 答题请留解题过程。

------------以下为试题区域，共三道大题，总分100分,考生请在答题纸上作答------------

**一、问答题（共 3 小题，共 20 分）**

1) (5分) For a matrix equation , where  is the solution vector, please tell (1) when there is no solution, (2) when there is a unique solution, and when there are infinitely many solutions.

2) (9分) Detail the following matrix factorization: 1) LU factorization; 2) Diagonalization; 3) QR decomposition.

3) (6分) What is the difference between orthogonal set and orthonormal set? What is orthogonal matrix?

**二、计算题（共 5 小题，共 60 分）**

1) (15分) Compute the reduced echelon form for the following matrix



Please point out the pivot positions. Is the above matrix invertible? If it is invertible, please compute its inverse.

2) (12分) Compute the determinant of the following matrices (6% for each):

, 

3) (8分) Are the following vectors linearly independent? Why?



4) (12分) Given the following matrix



Please

(a) Describe its column space and a basis for it. (3%)

(b) Compute the rank of matrix *A* (3%)

(c) Find an orthogonal basis for the column space of matrix *A*. (6%)

5) (13分) Given the following matrix



Please

(a) Compute its eigenvalues and eigenvectors . (6%)

(b) Is matrix *A* diagonalizable? If it is, please diagonalize it? (5%)

(c) What is the determinant value of *A?* (2%)

**三、证明题（共 3 小题，共 20 分）**

1.5CM

1) (8分) Suppose  is an orthogonal basis of a subspace *W* of . Then

(a) (2分) For any  in , what is its orthogonal projection onto *W*?

(b) (6分) Denote the orthogonal projection of  onto *W* by . Please prove that there exist  such that =+. Please also prove this kind of decomposition, namely reconstructing any vector by addition of two vectors in *W* and the complementary  respectively, is unique.

2) (6分) Let . Please prove that *W* is a subspace of .

3) (6分) Let  be a solution of matrix equation , where *A* is a matrix. Let  be a basis of the null space of matrix *A ,* and  is not zero. Please prove:

(a) (3分)  are linearly independent

(b) (3分)  are linearly independent